Pensieve header: Analysis of k=2 invariants in QU.

In[•]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Portfolio"];
<< KnotTheory`
<< "SL2PortfolioProgram.m"
```

```
Loading KnotTheory` version of January 20, 2015, 10:42:19.1122.
Read more at http://katlas.org/wiki/KnotTheory.
```

In[•]:=
```
OverbayP2Data = Get["C:\\drorbn\\AcademicPensieve\\People\\Overbay\\OverbayP2Data.m"];
```

In[•]:=
```
$p = 5; $k = 2; $U = QU;
```

In[•]:=
```
SCΘ[p_] := Collect[ CΘ@@𝒪CU[{y, a, x}, p] /. {CU → Times, γ | ħ → 1}, ϵ, Simplify];
SQΘ[p_] := Collect[ QΘ@@𝒪QU[{y, a, x}, p] /. {QU → Times, γ | ħ → 1}, ϵ, Simplify];
```

In[•]:=
```
𝔼[L_, Q_, P_]$k_ := 𝔼[L, Q, Series[Normal@P, {ϵ, 0, $k}]];
𝔼d_→r_[L_, Q_, P_]$k_ := 𝔼d→r @@ 𝔼[L, Q, P]$k;
𝔼3@𝔼[ω_, L_, Q_, Ps_] := CF /@ 𝔼[L, ω^-1 Q, ω^-1 (ω^-4 ϵ)^(-1+Range@Length@Ps .Ps)]$k;
𝔼4@𝔼[L_, Q_, P_] := Module[
   {ω = Normal[P]^-1 /. ϵ → 0, Ps = CoefficientList[P, ϵ]},
   CF /@ 𝔼[ω, L, ω Q, ω^(-3+4 Range@Length@Ps Ps)]];
𝔼3@𝔼sp___[as___] := 𝔼3@𝔼[as] /. 𝔼 → 𝔼sp;
𝔼4@𝔼sp___[as___] := 𝔼4@𝔼[as] /. 𝔼 → 𝔼sp;
```

In[•]:=
```
P[Knot[n_, k_]] := P[Knot[n, k]] = Module[{fname},
    fname = "../SL2Invariant/k=2/Data/" <> ToString[n] <> "_" <> ToString[k] <> ".m";
    Collect[𝔼3[Get[fname]⟦2, 2⟧]⟦3⟧ // Normal, ϵ, Simplify]
   ];
QP[K_Knot] := QP[K] = CF@P[K];
```

```
In[•]:= MatrixForm[AllKnots[{3, 7}] /.
  K_Knot :> {K, qp = Collect[QP@K /. {y → 0, a → -1/2}, {ϵ, a}, Factor]; ω = (qp /. ϵ → 0)^-1,
    P1 = Expand[(*T/(T-1)^2*) ω^3 Coefficient[qp, ϵ]], P2 = Expand[ω^5 Coefficient[qp, ϵ^2]],
    (*Factor[(P2-(P2/.T→-T))/(4(T+1/T))],*) Expand[2 P2 + (T + 1/T) ω P1],
    OP2 = K /. OverbayP2Data /. T → T^(1/2), {ω, ∂_T ω, P1, P2, OP2}} /. T → 1}]
```

Out[•]//MatrixForm=

Knot[3, 1]	$\frac{1-T+T^2}{T}$	$-2-\frac{1}{T^2}+\frac{2}{T}+2T-T^2$	
Knot[4, 1]	$-\frac{1-3T+T^2}{T}$	0	
Knot[5, 1]	$\frac{1-T+T^2-T^3+T^4}{T^2}$	$-6-\frac{2}{T^4}+\frac{4}{T^3}-\frac{5}{T^2}+\frac{6}{T}+6T-5T^2+4T^3-2T^4$	
Knot[5, 2]	$\frac{2-3T+2T^2}{T}$	$-18-\frac{5}{T^2}+\frac{14}{T}+14T-5T^2$	
Knot[6, 1]	$-\frac{(-2+T)(-1+2T)}{T}$	$-10-\frac{1}{T^2}+\frac{6}{T}+6T-T^2$	
Knot[6, 2]	$-\frac{1-3T+3T^2-3T^3+T^4}{T^2}$	$-16-\frac{1}{T^4}+\frac{6}{T^3}-\frac{13}{T^2}+\frac{16}{T}+16T-13T^2+6T^3-T^4$	
Knot[6, 3]	$\frac{1-3T+5T^2-3T^3+T^4}{T^2}$	0	
Knot[7, 1]	$\frac{1-T+T^2-T^3+T^4-T^5+T^6}{T^3}$	$-12-\frac{3}{T^6}+\frac{6}{T^5}-\frac{8}{T^4}+\frac{10}{T^3}-\frac{11}{T^2}+\frac{12}{T}+12T-11T^2+10T^3-8T^4+6T^5-3T^6$	686
Knot[7, 2]	$\frac{3-5T+3T^2}{T}$	$-60-\frac{14}{T^2}+\frac{44}{T}+44T-14T^2$	
Knot[7, 3]	$\frac{2-3T+3T^2-3T^3+2T^4}{T^2}$	$56+\frac{9}{T^4}-\frac{26}{T^3}+\frac{41}{T^2}-\frac{52}{T}-52T+41T^2-26T^3+9T^4$	
Knot[7, 4]	$\frac{4-7T+4T^2}{T}$	$112+\frac{24}{T^2}-\frac{80}{T}-80T+24T^2$	
Knot[7, 5]	$\frac{2-4T+5T^2-4T^3+2T^4}{T^2}$	$-114-\frac{9}{T^4}+\frac{34}{T^3}-\frac{70}{T^2}+\frac{102}{T}+102T-70T^2+34T^3-9T^4$	
Knot[7, 6]	$-\frac{1-5T+7T^2-5T^3+T^4}{T^2}$	$-78-\frac{1}{T^4}+\frac{10}{T^3}-\frac{36}{T^2}+\frac{66}{T}+66T-36T^2+10T^3-T^4$	
Knot[7, 7]	$\frac{1-5T+9T^2-5T^3+T^4}{T^2}$	$22+\frac{3}{T^2}-\frac{14}{T}-14T+3T^2$	

```
In[•]:= MatrixForm[
  mat = AllKnots[{3, 8}] /. K_Knot :> {K, qp = Collect[QP@K /. {y → 0}, {ϵ, a}, Factor];
    ω = (qp /. ϵ → 0)^-1, P1 = Expand[(*T/(T-1)^2*) ω^3 Coefficient[qp, ϵ] /. a → 0],
    P2 = Expand@Factor[ω^5 Coefficient[qp, ϵ^2] /. a → -1/2],
    OP2 = K /. OverbayP2Data /. T → T^(1/2), {ω, T ∂_T ω, T ∂_T (T ∂_T ω), P1, P2, OP2} /. T → -1}]
```

Out[•]//MatrixForm=

Knot[3, 1]	$\frac{1-T+T^2}{T}$	$-2-\frac{2}{T^2}+\frac{3}{T}+T$
Knot[4, 1]	$-\frac{1-3T+T^2}{T}$	$-\frac{1}{T^2}+\frac{3}{T}-3T+T^2$
Knot[5, 1]	$\frac{1-T+T^2-T^3+T^4}{T^2}$	$-6-\frac{4}{T^4}+\frac{7}{T^3}-\frac{8}{T^2}+\frac{8}{T}+4T-2T^2+T^3$
Knot[5, 2]	$\frac{2-3T+2T^2}{T}$	$-18-\frac{9}{T^2}+\frac{20}{T}+8T-T^2$
Knot[6, 1]	$-\frac{(-2+T)(-1+2T)}{T}$	$-10-\frac{5}{T^2}+\frac{16}{T}-4T+3T^2$
Knot[6, 2]	$-\frac{1-3T+3T^2-3T^3+T^4}{T^2}$	$-16-\frac{3}{T^4}+\frac{15}{T^3}-\frac{28}{T^2}+\frac{28}{T}+4T+2T^2-3T^3+T^4$
Knot[6, 3]	$\frac{1-3T+5T^2-3T^3+T^4}{T^2}$	$-\frac{2}{T^4}+\frac{9}{T^3}-\frac{19}{T^2}+\frac{18}{T}-18T+19T^2-9T^3+2T^4$
Knot[7, 1]	$\frac{1-T+T^2-T^3+T^4-T^5+T^6}{T^3}$	$-12-\frac{6}{T^6}+\frac{11}{T^5}-\frac{14}{T^4}+\frac{16}{T^3}-\frac{16}{T^2}+\frac{15}{T}+9T-6T^2+4T^3-2T^4$
Knot[7, 2]	$\frac{3-5T+3T^2}{T}$	$-60-\frac{23}{T^2}+\frac{59}{T}+29T-5T^2$
	$\frac{2-3T+3T^2-3T^3+2T^4}{T^2}$	$\frac{1}{T^4}-\frac{8}{T^3}+\frac{20}{T^2}-\frac{37}{T}$

Knot[7, 3]	$\frac{2-3\,T+3\,T^2-3\,T^3+2\,T^4}{T^2}$	$56+\frac{1}{T^4}-\frac{8}{T^3}+\frac{20}{T^2}-\frac{37}{T}-67\,T+62\,T^2-44\,T^3+17\,T^4$
Knot[7, 4]	$\frac{4-7\,T+4\,T^2}{T}$	$112+\frac{8}{T^2}-\frac{52}{T}-108\,T+40\,T^2$
Knot[7, 5]	$\frac{2-4\,T+5\,T^2-4\,T^3+2\,T^4}{T^2}$	$-114-\frac{17}{T^4}+\frac{58}{T^3}-\frac{106}{T^2}+\frac{130}{T}+74\,T-34\,T^2+10\,T^3-T^4$
Knot[7, 6]	$-\frac{1-5\,T+7\,T^2-5\,T^3+T^4}{T^2}$	$-78-\frac{3}{T^4}+\frac{25}{T^3}-\frac{75}{T^2}+\frac{106}{T}+26\,T+3\,T^2-5\,T^3+T^4$
Knot[7, 7]	$\frac{1-5\,T+9\,T^2-5\,T^3+T^4}{T^2}$	$22-\frac{2}{T^4}+\frac{15}{T^3}-\frac{40}{T^2}+\frac{36}{T}-64\,T+46\,T^2-15\,T^3+2\,T^4$
Knot[8, 1]	$-\frac{3-7\,T+3\,T^2}{T}$	$-42-\frac{14}{T^2}+\frac{47}{T}+5\,T+4\,T^2$
Knot[8, 2]	$-\frac{1-3\,T+3\,T^2-3\,T^3+3\,T^4-3\,T^5+T^6}{T^3}$	$-50-\frac{5}{T^6}+\frac{27}{T^5}-\frac{58}{T^4}+\frac{76}{T^3}-\frac{80}{T^2}+\frac{71}{T}+29\,T-14\,T^2+4\,T^3+2\,T^4-3$
Knot[8, 3]	$-\frac{4-9\,T+4\,T^2}{T}$	$-\frac{16}{T^2}+\frac{36}{T}-36\,T+16\,T^2$
Knot[8, 4]	$-\frac{2-5\,T+5\,T^2-5\,T^3+2\,T^4}{T^2}$	$-20-\frac{11}{T^4}+\frac{44}{T^3}-\frac{70}{T^2}+\frac{59}{T}-11\,T+20\,T^2-16\,T^3+5\,T^4$
Knot[8, 5]	$-\frac{\left(1-T+T^2\right)\left(1-2\,T+T^2-2\,T^3+T^4\right)}{T^3}$	$92-\frac{1}{T^6}+\frac{3}{T^5}-\frac{3}{T^4}-\frac{3}{T^3}+\frac{21}{T^2}-\frac{53}{T}-123\,T+129\,T^2-105\,T^3+65\,T^4-$
Knot[8, 6]	$-\frac{2-6\,T+7\,T^2-6\,T^3+2\,T^4}{T^2}$	$-120-\frac{13}{T^4}+\frac{66}{T^3}-\frac{137}{T^2}+\frac{162}{T}+54\,T-9\,T^2-6\,T^3+3\,T^4$
Knot[8, 7]	$\frac{1-3\,T+5\,T^2-5\,T^3+5\,T^4-3\,T^5+T^6}{T^3}$	$50-\frac{2}{T^6}+\frac{9}{T^5}-\frac{19}{T^4}+\frac{24}{T^3}-\frac{18}{T^2}-\frac{7}{T}-93\,T+112\,T^2-96\,T^3+57\,T^4-2$
Knot[8, 8]	$\frac{\left(2-2\,T+T^2\right)\left(1-2\,T+2\,T^2\right)}{T^2}$	$56-\frac{7}{T^4}+\frac{30}{T^3}-\frac{51}{T^2}+\frac{22}{T}-110\,T+93\,T^2-42\,T^3+9\,T^4$
Knot[8, 9]	$-\frac{\left(-1+T-2\,T^2+T^3\right)\left(-1+2\,T-T^2+T^3\right)}{T^3}$	$-\frac{3}{T^6}+\frac{15}{T^5}-\frac{38}{T^4}+\frac{66}{T^3}-\frac{77}{T^2}+\frac{53}{T}-53\,T+77\,T^2-66\,T^3+38\,T^4-15$
Knot[8, 10]	$\frac{\left(1-T+T^2\right)^3}{T^3}$	$82-\frac{2}{T^6}+\frac{9}{T^5}-\frac{22}{T^4}+\frac{33}{T^3}-\frac{26}{T^2}-\frac{15}{T}-141\,T+154\,T^2-117\,T^3+62\,T^4-$
Knot[8, 11]	$-\frac{(-2+T)\,(-1+2\,T)\left(1-T+T^2\right)}{T^2}$	$-166-\frac{13}{T^4}+\frac{76}{T^3}-\frac{177}{T^2}+\frac{223}{T}+69\,T-7\,T^2-8\,T^3+3\,T^4$
Knot[8, 12]	$\frac{1-7\,T+13\,T^2-7\,T^3+T^4}{T^2}$	$-\frac{2}{T^4}+\frac{21}{T^3}-\frac{75}{T^2}+\frac{98}{T}-98\,T+75\,T^2-21\,T^3+2\,T^4$
Knot[8, 13]	$\frac{2-7\,T+11\,T^2-7\,T^3+2\,T^4}{T^2}$	$68-\frac{7}{T^4}+\frac{36}{T^3}-\frac{70}{T^2}+\frac{39}{T}-143\,T+116\,T^2-48\,T^3+9\,T^4$
Knot[8, 14]	$-\frac{2-8\,T+11\,T^2-8\,T^3+2\,T^4}{T^2}$	$-250-\frac{13}{T^4}+\frac{86}{T^3}-\frac{226}{T^2}+\frac{314}{T}+106\,T-10\,T^2-10\,T^3+3\,T^4$
Knot[8, 15]	$\frac{\left(1-T+T^2\right)\left(3-5\,T+3\,T^2\right)}{T^2}$	$-520-\frac{39}{T^4}+\frac{178}{T^3}-\frac{399}{T^2}+\frac{556}{T}+332\,T-139\,T^2+34\,T^3-3\,T$
Knot[8, 16]	$\frac{1-4\,T+8\,T^2-9\,T^3+8\,T^4-4\,T^5+T^6}{T^3}$	$-142-\frac{4}{T^6}+\frac{28}{T^5}-\frac{94}{T^4}+\frac{191}{T^3}-\frac{260}{T^2}+\frac{242}{T}+26\,T+44\,T^2-55\,T^3+34\,T^4-$
Knot[8, 17]	$-\frac{1-4\,T+8\,T^2-11\,T^3+8\,T^4-4\,T^5+T^6}{T^3}$	$-\frac{3}{T^6}+\frac{20}{T^5}-\frac{64}{T^4}+\frac{129}{T^3}-\frac{168}{T^2}+\frac{124}{T}-124\,T+168\,T^2-129\,T^3+64\,T^4-$
Knot[8, 18]	$-\frac{\left(1-3\,T+T^2\right)\left(1-T+T^2\right)^2}{T^3}$	$-\frac{3}{T^6}+\frac{25}{T^5}-\frac{90}{T^4}+\frac{189}{T^3}-\frac{250}{T^2}+\frac{185}{T}-185\,T+250\,T^2-189\,T^3+90\,T^4-$
Knot[8, 19]	$\frac{\left(1-T+T^2\right)\left(1-T^2+T^4\right)}{T^3}$	$6-\frac{1}{T^5}+\frac{1}{T^4}+\frac{1}{T^3}-\frac{3}{T^2}-\frac{1}{T}-3\,T-7\,T^2+7\,T^3+5\,T^4-11\,T^5+$
Knot[8, 20]	$\frac{\left(1-T+T^2\right)^2}{T^2}$	$-16-\frac{2}{T^4}+\frac{6}{T^3}-\frac{14}{T^2}+\frac{20}{T}+4\,T+6\,T^2-6\,T^3+2\,T^4$
Knot[8, 21]	$-\frac{\left(1-3\,T+T^2\right)\left(1-T+T^2\right)}{T^2}$	$-72-\frac{3}{T^4}+\frac{22}{T^3}-\frac{59}{T^2}+\frac{84}{T}+36\,T-7\,T^2-2\,T^3+T^4$

```
In[•]:= MatrixForm[mat = AllKnots[{3, 8}] /.
    K_Knot :> (K; qp = Collect[QP@K /. {y → 0}, {ϵ, a}, Factor]; ω = (qp /. ϵ → 0)^-1;
      P1 = Expand[(* T/(T-1)^2 *) ω^3 Coefficient[qp, ϵ] /. a → 0];
      P2 = Expand@Factor[ω^5 Coefficient[qp, ϵ^2] /. a → -1/2];
      OP2 = K /. OverbayP2Data /. T → T^(1/2);
      {q1 = ω, q2 = T ∂_T (T ∂_T ω), q3 = P1, q4 = T ∂_T P1,
        q5 = T ∂_T (T ∂_T P1), q1 q2, q1 q3, q2 q3, OP2, P2} /. T → -2)]
  Dimensions[mat]
  MatrixRank[mat]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
-\frac{7}{2} & -\frac{5}{2} & -6 & \frac{1}{2} & -\frac{11}{2} & \frac{35}{4} & 21 & 15 & -\frac{23}{4} & \frac{2021}{32} \\
\frac{11}{2} & \frac{5}{2} & \frac{33}{4} & 16 & \frac{39}{2} & \frac{55}{4} & \frac{363}{8} & \frac{165}{8} & \frac{77}{4} & \frac{77}{4} \\
\frac{31}{4} & \frac{39}{2} & -\frac{297}{8} & -\frac{291}{8} & -\frac{1087}{8} & \frac{1209}{8} & -\frac{9207}{32} & -\frac{11583}{16} & \frac{598809}{256} & \frac{88779}{16} \\
-8 & -5 & -\frac{201}{4} & -\frac{19}{2} & -51 & 40 & 402 & \frac{1005}{4} & \frac{8339}{16} & \frac{46355}{16} \\
10 & 5 & \frac{3}{4} & \frac{85}{2} & 43 & 50 & \frac{15}{2} & \frac{15}{4} & -\frac{7133}{16} & \frac{16267}{16} \\
-\frac{59}{4} & -\frac{49}{2} & \frac{15}{16} & \frac{1427}{8} & \frac{3473}{8} & \frac{2891}{8} & -\frac{885}{64} & -\frac{735}{32} & -\frac{198145}{64} & \frac{8220817}{512} \\
\frac{67}{4} & \frac{49}{2} & 201 & \frac{4435}{8} & \frac{11679}{8} & \frac{3283}{8} & \frac{13467}{4} & \frac{9849}{2} & \frac{143983}{64} & \frac{143983}{64} \\
-\frac{127}{8} & -\frac{741}{8} & -\frac{2637}{16} & -\frac{13527}{32} & -\frac{57007}{32} & \frac{94107}{64} & \frac{334899}{128} & \frac{1954017}{128} & \frac{571757703}{4096} & \frac{1848272061}{8192} \\
-\frac{25}{2} & -\frac{15}{2} & -\frac{693}{4} & -57 & -\frac{381}{2} & \frac{375}{4} & \frac{17325}{8} & \frac{10395}{8} & \frac{84111}{8} & \frac{455097}{16} \\
19 & \frac{83}{2} & \frac{17385}{16} & \frac{10969}{4} & \frac{17389}{2} & \frac{1577}{2} & \frac{330315}{16} & \frac{1442955}{32} & \frac{32612987}{256} & \frac{65700803}{256} \\
-17 & -10 & 516 & 506 & 890 & 170 & -8772 & -5160 & \frac{128221}{4} & \frac{367513}{4} \\
\frac{47}{2} & 44 & -\frac{9501}{16} & -580 & -\frac{7685}{4} & 1034 & -\frac{446547}{32} & -\frac{104511}{4} & \frac{76685911}{256} & \frac{153545857}{256} \\
-\frac{95}{4} & -\frac{59}{2} & -\frac{2193}{16} & \frac{2053}{8} & \frac{3623}{8} & \frac{5605}{8} & \frac{208335}{64} & \frac{129387}{32} & -\frac{14149}{8} & \frac{70328789}{512} \\
\frac{103}{4} & \frac{59}{2} & 456 & \frac{8225}{8} & \frac{19033}{8} & \frac{6077}{8} & 11742 & 13452 & \frac{128735}{64} & \frac{3217381}{128} \\
\frac{29}{2} & \frac{15}{2} & -63 & \frac{105}{2} & \frac{33}{2} & \frac{435}{4} & -\frac{1827}{2} & -\frac{945}{2} & \frac{69}{16} & \frac{431571}{32} \\
\frac{251}{8} & \frac{1053}{8} & -\frac{4707}{64} & \frac{13587}{16} & \frac{139619}{32} & \frac{264303}{64} & -\frac{1181457}{512} & -\frac{4956471}{512} & \frac{2092334229}{4096} & \frac{2721395205}{2048} \\
19 & 10 & 114 & 226 & 294 & 190 & 2166 & 1140 & -\frac{48431}{4} & \frac{48431}{4} \\
-26 & -\frac{93}{2} & \frac{3789}{16} & \frac{3879}{4} & 2614 & 1209 & -\frac{49257}{8} & -\frac{352377}{32} & -\frac{8968917}{256} & \frac{21159051}{256} \\
\frac{287}{8} & \frac{1073}{8} & \frac{252789}{64} & \frac{226579}{16} & \frac{1909599}{32} & \frac{307951}{64} & \frac{72550443}{512} & \frac{271242597}{512} & \frac{3253152509}{4096} & \frac{1091902663}{512} \\
-\frac{61}{2} & -49 & -\frac{4677}{16} & \frac{667}{2} & \frac{2571}{4} & \frac{2989}{2} & \frac{285297}{32} & \frac{229173}{16} & \frac{24261571}{256} & \frac{146228509}{256} \\
-\frac{307}{8} & -\frac{1093}{8} & \frac{6573}{2} & \frac{382427}{32} & \frac{1581739}{32} & \frac{335551}{64} & -\frac{2017911}{16} & -\frac{7184289}{16} & -\frac{177292237}{1024} & \frac{5992743721}{8192} \\
\frac{65}{2} & 49 & \frac{17601}{16} & \frac{5195}{2} & \frac{27733}{4} & \frac{3185}{2} & \frac{1144065}{32} & \frac{862449}{16} & -\frac{11420497}{256} & \frac{31851953}{256} \\
\frac{323}{8} & \frac{1093}{8} & \frac{138567}{64} & \frac{67135}{8} & \frac{1104027}{32} & \frac{353039}{64} & \frac{44757141}{512} & \frac{151453731}{512} & \frac{15583135}{1024} & \frac{15583135}{1024} \\
-\frac{343}{8} & -\frac{1113}{8} & \frac{61299}{16} & \frac{422751}{32} & \frac{1695111}{32} & \frac{381759}{64} & -\frac{21025557}{128} & -\frac{68225787}{128} & -\frac{58034571}{256} & \frac{9194017245}{8192} \\
-35 & -\frac{103}{2} & -\frac{6177}{16} & \frac{1687}{4} & 707 & \frac{3605}{2} & \frac{216195}{16} & \frac{636231}{32} & \frac{29865947}{256} & \frac{230344547}{256} \\
\frac{139}{4} & \frac{69}{2} & \frac{1251}{2} & \frac{12183}{8} & \frac{26163}{8} & \frac{9591}{8} & \frac{173889}{8} & \frac{86319}{4} & \frac{833583}{8} & \frac{833583}{8} \\
37 & \frac{103}{2} & \frac{20865}{16} & \frac{12047}{4} & 7765 & \frac{3811}{2} & \frac{772005}{16} & \frac{2149095}{32} & -\frac{14802341}{256} & \frac{48379747}{256} \\
-\frac{79}{2} & -54 & -\frac{9585}{16} & \frac{891}{2} & \frac{2493}{4} & 2133 & \frac{757215}{32} & \frac{258795}{8} & \frac{63215379}{256} & \frac{395848197}{256} \\
\frac{175}{4} & 71 & -\frac{39399}{16} & -2230 & -\frac{28081}{4} & \frac{12425}{4} & -\frac{6894825}{64} & -\frac{2797329}{16} & \frac{1105994203}{256} & \frac{4158176531}{512} \\
-\frac{433}{8} & -\frac{1289}{8} & \frac{20181}{16} & \frac{54679}{8} & 26810 & \frac{558137}{64} & -\frac{8738373}{128} & -\frac{26013309}{128} & -\frac{107890943}{256} & \frac{24033564713}{8192} \\
\frac{449}{8} & \frac{1289}{8} & \frac{235725}{64} & \frac{427193}{32} & \frac{817017}{16} & \frac{578761}{64} & \frac{105840525}{512} & \frac{303849525}{512} & \frac{441280343}{1024} & \frac{441280343}{1024} \\
\frac{539}{8} & \frac{1465}{8} & \frac{328251}{64} & \frac{290129}{16} & \frac{2152437}{32} & \frac{789635}{64} & \frac{176927289}{512} & \frac{480887715}{512} & \frac{304228309}{256} & \frac{304228309}{256} \\
-\frac{91}{8} & -\frac{721}{8} & \frac{23799}{32} & \frac{133343}{32} & \frac{745349}{32} & \frac{65611}{64} & -\frac{2165709}{256} & -\frac{17159079}{256} & \frac{195680651}{4096} & \frac{597881161}{8192} \\
\frac{49}{4} & 22 & \frac{525}{8} & \frac{1327}{4} & \frac{3997}{4} & \frac{539}{2} & \frac{25725}{32} & \frac{5775}{4} & -\frac{243187}{64} & \frac{59339}{64} \\
-\frac{77}{4} & -27 & -\frac{3195}{16} & \frac{129}{2} & \frac{349}{4} & \frac{2079}{4} & \frac{246015}{64} & \frac{86265}{16} & \frac{3566727}{128} & \frac{57862845}{512}
\end{pmatrix}$$

Out[•]= {35, 10}

Out[•]= 10

In[•]:=
```
NullSpace[mat]
```

Out[•]= {{0, 0, 0, -1, 1, 0, 0, 0, 0}}

In[•]:=
```
MatrixForm[mat = AllKnots[{3, 8}] /.
    K_Knot :> (K; qp = Collect[QP@K /. {y → 0}, {ϵ, a}, Factor]; ω = (qp /. ϵ → 0)^-1;
      P1 = Expand[(* T/(T-1)^2 *) ω^3 Coefficient[qp, ϵ] /. a → 0];
      P2 = Expand@Factor[ω^5 Coefficient[qp, ϵ^2] /. a → -1/2];
      OP2 = K /. OverbayP2Data /. T → T^(1/2);
      {q1 = ω, q2 = T ∂_T (T ∂_T ω), q3 = P1, q4 = T ∂_T P1, q1 q2, q1 q3, q2 q3, OP2, P2} /. T → -2)]
Dimensions[mat]
MatrixRank[mat]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
-\frac{7}{2} & -\frac{5}{2} & -6 & \frac{1}{2} & \frac{35}{4} & 21 & 15 & -\frac{23}{4} & \frac{2021}{32} \\
\frac{11}{2} & \frac{5}{2} & \frac{33}{4} & 16 & \frac{55}{4} & \frac{363}{8} & \frac{165}{8} & \frac{77}{4} & \frac{77}{4} \\
\frac{31}{4} & \frac{39}{2} & -\frac{297}{8} & -\frac{291}{8} & \frac{1209}{8} & -\frac{9207}{32} & -\frac{11583}{16} & \frac{598809}{256} & \frac{88779}{16} \\
-8 & -5 & -\frac{201}{4} & -\frac{19}{2} & 40 & 402 & \frac{1005}{4} & \frac{8339}{16} & \frac{46355}{16} \\
10 & 5 & \frac{3}{4} & \frac{85}{2} & 50 & \frac{15}{2} & \frac{15}{4} & -\frac{7133}{16} & \frac{16267}{16} \\
-\frac{59}{4} & -\frac{49}{2} & \frac{15}{16} & \frac{1427}{8} & \frac{2891}{8} & -\frac{885}{64} & -\frac{735}{32} & -\frac{198145}{64} & \frac{8220817}{512} \\
\frac{67}{4} & \frac{49}{2} & 201 & \frac{4435}{8} & \frac{3283}{8} & \frac{13467}{4} & \frac{9849}{2} & \frac{143983}{64} & \frac{143983}{64} \\
-\frac{127}{8} & -\frac{741}{8} & -\frac{2637}{16} & -\frac{13527}{32} & \frac{94107}{64} & \frac{334899}{128} & \frac{1954017}{128} & \frac{571757703}{4096} & \frac{1848272061}{8192} \\
-\frac{25}{2} & -\frac{15}{2} & -\frac{693}{4} & -57 & \frac{375}{4} & \frac{17325}{8} & \frac{10395}{8} & \frac{84111}{8} & \frac{455097}{16} \\
19 & \frac{83}{2} & \frac{17385}{16} & \frac{10969}{4} & \frac{1577}{2} & \frac{330315}{16} & \frac{1442955}{32} & \frac{32612987}{256} & \frac{65700803}{256} \\
-17 & -10 & 516 & 506 & 170 & -8772 & -5160 & \frac{128221}{4} & \frac{367513}{4} \\
\frac{47}{2} & 44 & -\frac{9501}{16} & -580 & 1034 & -\frac{446547}{32} & -\frac{104511}{4} & \frac{76685911}{256} & \frac{153545857}{256} \\
-\frac{95}{4} & -\frac{59}{2} & -\frac{2193}{16} & \frac{2053}{8} & \frac{5605}{8} & \frac{208335}{64} & \frac{129387}{32} & -\frac{14149}{8} & \frac{70328789}{512} \\
\frac{103}{4} & \frac{59}{2} & 456 & \frac{8225}{8} & \frac{6077}{8} & 11742 & 13452 & \frac{128735}{64} & \frac{3217381}{128} \\
\frac{29}{2} & \frac{15}{2} & -63 & \frac{105}{2} & \frac{435}{4} & -\frac{1827}{2} & -\frac{945}{2} & \frac{69}{16} & \frac{431571}{32} \\
\frac{251}{8} & \frac{1053}{8} & -\frac{4707}{64} & \frac{13587}{16} & \frac{264303}{64} & -\frac{1181457}{512} & -\frac{4956471}{512} & \frac{2092334229}{4096} & \frac{2721395205}{2048} \\
19 & 10 & 114 & 226 & 190 & 2166 & 1140 & -\frac{48431}{4} & \frac{48431}{4} \\
-26 & -\frac{93}{2} & \frac{3789}{16} & \frac{3879}{4} & 1209 & -\frac{49257}{8} & -\frac{352377}{32} & -\frac{8968917}{256} & \frac{21159051}{256} \\
\frac{287}{8} & \frac{1073}{8} & \frac{252789}{64} & \frac{226579}{16} & \frac{307951}{64} & \frac{72550443}{512} & \frac{271242597}{512} & \frac{3253152509}{4096} & \frac{1091902663}{512} \\
-\frac{61}{2} & -49 & -\frac{4677}{16} & \frac{667}{2} & \frac{2989}{2} & \frac{285297}{32} & \frac{229173}{16} & \frac{24261571}{256} & \frac{146228509}{256} \\
-\frac{307}{8} & -\frac{1093}{8} & \frac{6573}{2} & \frac{382427}{32} & \frac{335551}{64} & -\frac{2017911}{16} & -\frac{7184289}{16} & -\frac{177292237}{1024} & \frac{5992743721}{8192} \\
\frac{65}{2} & 49 & \frac{17601}{16} & \frac{5195}{2} & \frac{3185}{2} & \frac{1144065}{32} & \frac{862449}{16} & -\frac{11420497}{256} & \frac{31851953}{256} \\
\frac{323}{8} & \frac{1093}{8} & \frac{138567}{64} & \frac{67135}{8} & \frac{353039}{64} & \frac{44757141}{512} & \frac{151453731}{512} & \frac{15583135}{1024} & \frac{15583135}{1024} \\
-\frac{343}{8} & -\frac{1113}{8} & \frac{61299}{16} & \frac{422751}{32} & \frac{381759}{64} & -\frac{21025557}{128} & -\frac{68225787}{128} & -\frac{58034571}{256} & \frac{9194017245}{8192} \\
-35 & -\frac{103}{2} & -\frac{6177}{16} & \frac{1687}{4} & \frac{3605}{2} & \frac{216195}{16} & \frac{636231}{32} & \frac{29865947}{256} & \frac{230344547}{256} \\
\frac{139}{4} & \frac{69}{2} & \frac{1251}{2} & \frac{12183}{8} & \frac{9591}{8} & \frac{173889}{8} & \frac{86319}{4} & \frac{833583}{8} & \frac{833583}{8} \\
37 & \frac{103}{2} & \frac{20865}{16} & \frac{12047}{4} & \frac{3811}{2} & \frac{772005}{16} & \frac{2149095}{32} & -\frac{14802341}{256} & \frac{48379747}{256} \\
-\frac{79}{2} & -54 & -\frac{9585}{16} & \frac{891}{2} & 2133 & \frac{757215}{32} & \frac{258795}{8} & \frac{63215379}{256} & \frac{395848197}{256} \\
\frac{175}{4} & 71 & -\frac{39399}{16} & -2230 & \frac{12425}{4} & -\frac{6894825}{64} & -\frac{2797329}{16} & \frac{1105994203}{256} & \frac{4158176531}{512} \\
-\frac{433}{8} & -\frac{1289}{8} & \frac{20181}{16} & \frac{54679}{8} & \frac{558137}{64} & -\frac{8738373}{128} & -\frac{26013309}{128} & -\frac{107890943}{256} & \frac{24033564713}{8192} \\
\frac{449}{8} & \frac{1289}{8} & \frac{235725}{64} & \frac{427193}{32} & \frac{578761}{64} & \frac{105840525}{512} & \frac{303849525}{512} & \frac{441280343}{1024} & \frac{441280343}{1024} \\
\frac{539}{8} & \frac{1465}{8} & \frac{328251}{64} & \frac{290129}{16} & \frac{789635}{64} & \frac{176927289}{512} & \frac{480887715}{512} & \frac{304228309}{256} & \frac{304228309}{256} \\
-\frac{91}{8} & -\frac{721}{8} & \frac{23799}{32} & \frac{133343}{32} & \frac{65611}{64} & -\frac{2165709}{256} & -\frac{17159079}{256} & \frac{195680651}{4096} & \frac{597881161}{8192} \\
\frac{49}{4} & 22 & \frac{525}{8} & \frac{1327}{4} & \frac{539}{2} & \frac{25725}{32} & \frac{5775}{4} & -\frac{243187}{64} & \frac{59339}{64} \\
-\frac{77}{4} & -27 & -\frac{3195}{16} & \frac{129}{2} & \frac{2079}{4} & \frac{246015}{64} & \frac{86265}{16} & \frac{3566727}{128} & \frac{57862845}{512}
\end{pmatrix}$$

Out[•]= {35, 9}

Out[•]= 9